Robust weighted composite indicators by means of frontier methods: an application to European infrastructure endowment

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Abstract

The aim of this paper is to more systematically and consistently present some recent contributions that have attempted to combine two different fields: the construction of composite indicators (CI's) and the measurement of productive efficiency by means of frontier techniques. In particular, we have proposed to correct the classical Benefit of Doubt approach index by means a non-compensatory approach and to introduce a more robust estimator using order-m tecniques. Suggested methods have been tested with reference to infrastructure endowment in European regions.

Keywords: *Composite Indicators; Data Envelopment Analysis, Order-m frontier, Conditional order-m frontier.*

JEL classification: *C43, C61.*

1 Introduction

The aim of this paper is to more systematically and consistently present some recent contributions that have attempted to combine two different fields: the construction of composite indicators (CI's) and the measurement of productive efficiency by means of frontier techniques.

Both fields, in recent years, have made considerable theoretical and applied progress (see e.g. Witte and Rogge (2009)) and improvements, but, in our consideration, has not yet fully studied the possible uses of the frontier techniques in the construction of CI's.

In particular, after having systematized the different theoretical approaches, we have proposed some improvements both in the robustness field and with respect to non-compensatory approaches.

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1.1 Composite indicators: history and methods

Composite indicators which compare country performance are increasingly recognized as a useful tool in policy analysis and public communication, for a variety of policy matters such as industrial competitiveness, sustainable development, quality of life assessment, globalization and innovation. They provide simple comparisons of countries that can be used to illustrate complex and sometimes elusive issues in wide ranging fields, e.g., the environment, economy, social or technological development. These indicators often seem easier to interpret by the general public finding a common trend in many separate indicators and have proven useful in benchmarking country performance.

Along such lines the Joint Research Centre of European Commission asserts that *"no uniformly agreed methodology exists to weight individual indicators before aggregating them into a composite indicator"*[1].

A much wider ranging literature is found for the *aggregation* methods than the one regarding *weight* systems; however, the two aspects are related and interwoven and often lead to the same solutions.

Among many approaches proposed in the literature[2], two relate to our purposes: the first one, called BoD based on Data Envelopment Analysis (DEA) methods and the second one named MPCV[3] proposed in 2010 by DeMuro et al. (2010).

The first approach (DEA techniques) have been used, among others, for the European labor market analysis Storrie and Bjurek (2000), for social inclusion policies at EU level Cherchye et al. (2004) and for the internal market policies Cherchye et al. (2005). Similarly, some authors have suggested applying DEA techniques to the Human Development Index (HDI) Mahlberg and Obersteiner (2001), Despotis (2005a), Despotis (2005b), Cherchye et al. (2008)).

The second approach, starting with a linear aggregation, emphasizes the non-compensability between indicators, introducing penalties for units that present an *unbalanced* basic indicators' set.

For a greater clarity, we have simulated, for each method proposed, the calculation of the composite indicators for two basic indicators $I_1, I_2 \in [0, 1]$; in Figure 1, for example, the CI's value by MPCV method that penalize units with an higher values for I_1 and a lower I_2 value (and viceversa), by rewarding the *"balanced"* sets is represented.

[1] `http://composite-indicators.jrc.ec.europa.eu/S6_weighting.htm`

[2] For a complete review, please see Nardo et al. (2005) and Freudenberg (2003) for major applications and papers.

[3] Metodo di Penalita' per Coefficiente di Variazione; in English: Method of Penalty for Coefficient of Variation.

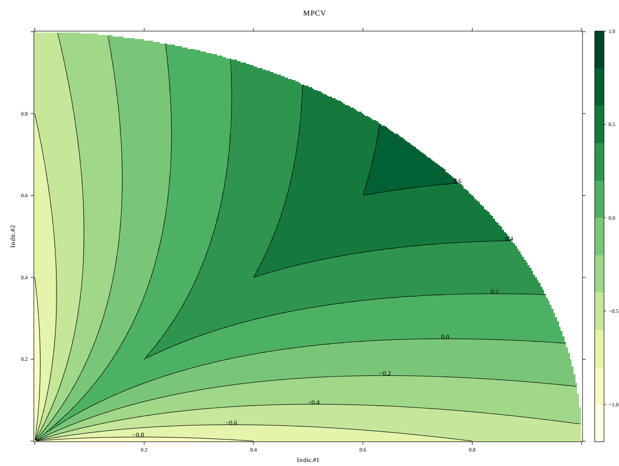

Figure 1: MPCV distribution

Several steps are involved in creating composite indicators: *investigate the structure* of simple indicators by means of multivariate statistic, handling the problem of *missing data* that can be missing either in a random or in a non-random fashion, bringing the indicators to the same unit to avoid adding up apples and pears by *normalization* and finally selecting an appropriate *weighting and aggregation* model. (for a complete explanation of every step, please see Nardo et al. (2005))

Our analysis will only focus on the *weighting and aggregation* phase, that in the field of CI's is of great importance and has yet to be really fully developed.

The present paper is presented as follows: in the second section a basic productive efficiency framework is briefly outlined; in the third, fourth and fifth section BoD, robust BoD and Conditional robust BoD are introduced; the application to European infrastructure is presented in the sixth section and then the conclusive observations are presented in the seventh one.

2 Basic productive efficiency concepts

In the last decades productive efficiency has been widely analyzed[4] and estimated more often by using two different approaches: parametric and nonparametric techniques.

The first one specifies a priori a functional form with constant parameters, which are estimated with statistical and econometric methods. So for each observation, the measurement of efficiency in terms of input, both output, is

[4]For a complete survey, please see Fried and Lovell (2008).

calculated with reference to the function estimated, which represents the frontier of production technique under consideration. Evidently, the measure thus obtained depends on the functional form specified in advance.

Consider a production technology where the activity of each DMU's is characterized by a set of inputs $x \in \mathbb{R}_+^p$ used to produce a set of outputs $y \in \mathbb{R}_+^q$. The production set is the set of technically feasible combinations of (x, y):

$$\Psi = \{(x, y) \in \mathbb{R}^{p+q} | x \text{ can produce } y\} \tag{1}$$

Ψ is the so called the support of $H(x, y)$.

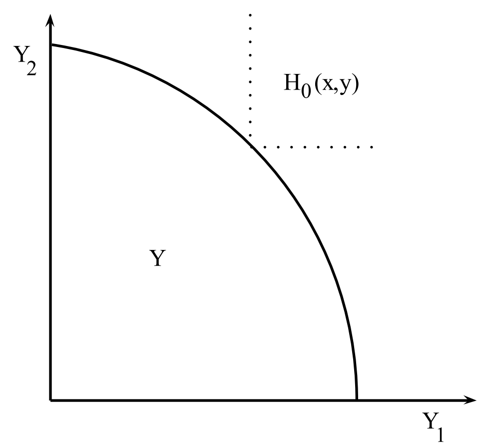

Figure 2: Support of $H_0(x, y)$

Some assumptions are usually made on this set, such as the free disposability of inputs and outputs, meaning that if $(x, y) \in \Psi$, then $(x', y') \in \Psi$, as soon as $x' \geq x$ and $y' \leq y$.

The Farrell-Debreu efficiency scores for a given production scenario $(x, y) \in \Psi$, are defined as:

$$\text{Input oriented} \quad \theta(x, y) = \inf\{\theta | (\theta x, y) \in \Psi\} \tag{2}$$

$$\text{Output oriented} \quad \lambda(x, y) = \sup\{\lambda | (x, \lambda y) \in \Psi\} \tag{3}$$

In practice Ψ is unknown and so has to be estimated from a random sample of production units $\chi = \{(X_i, Y_i) | i = 1, ...n\}$, where we assume that $Prob((X_i, Y_i) \in \Psi)$ (so called deterministic frontier models).

The matter is related to the problem of estimating the support of the random variable (X, Y) where Ψ is supposed to be compact.

The most popular nonparametric estimators are based on the Farrel-Debreu envelopment ideas; in this paper we present the input oriented approach.

The Free Disposal Hull (FDH) estimator Deprins et al. (1984) is provided by the free disposal hull of the sample points X:

$$\widehat{\Psi}_{FDH} = \{(x, y) \in \mathbb{R}^{p+q} | y < Y_i, x \geq X_i,\ i = 1, ..., n\} \tag{4}$$

The FDH efficiency scores are obtained by plugging $\widehat{\Psi}_{FDH}$ in equations 2 and 3 in place of the unknown Ψ.

If Ψ is convex, take the convex hull of $\widehat{\Psi}_{FDH}$:

$$\begin{aligned} \widehat{\Psi}_{DEA} = \{(x, y) \in \mathbb{R}^{p+q} | y < \sum_{i=1}^{n} \gamma_i yi \text{ for } (\gamma_1, ..\gamma_n) \\ \text{such that } \sum_{i=1}^{n} \gamma_i = 1; \gamma_i \geq 0, i = 1, ..., n\} \end{aligned} \tag{5}$$

3 Benefit of the Doubt approach (DEA application to CI)

"The Benefit of the Doubt approach is formally tantamount to the original input-oriented CCR-DEA model of Charnes et al. Charnes et al. (1978), with all questionnaire items considered as outputs and a dummy input equal to one for all observations" Witte and Rogge (2009).

The Farrell-Debreu efficiency scores (input oriented) for a given production scenario $(x, y) \in \Psi$ when x is constant and equal to 1 may be defined as:

$$\theta(x, y) = \inf\{\theta | (\theta, y) \in \Psi\} \tag{6}$$

So the Free Disposal Hull (FDH) estimator is provided by the particular free disposal hull of the sample points:

$$\widehat{\Psi}_{FDH} = \{(\mathbb{1}, y) \in \mathbb{R}^{1+q} | y < Y_i, i = 1, ..., n\} \tag{7}$$

if Ψ is convex, take the convex hull of $\widehat{\Psi}_{FDH}$ called Benefit of Doubt (BoD) in accordance with Cherchye and Kuosmanen (2002):

$$\begin{aligned} \widehat{\Psi}_{BoD} = \{(\mathbb{1}, y) \in \mathbb{R}^{1+q} | y < \sum_{i=1}^{n} \gamma_i yi \text{ for } (\gamma_1, ..\gamma_n) \\ \text{such that } \sum_{i=1}^{n} \gamma_i = 1; \gamma_i \geq 0, i = 1, ..., n\} \end{aligned} \tag{8}$$

In Figure 3 we plot the BoD distribution; it may be readily noted that the composite score depends exclusively on the frontier's distance and not, contrary to the MPCV, on the relationship *between* simple indicators. In subsection 3.2 we join BoD and MPCV methods to overcome, in a non compensatory framework, this drawback.

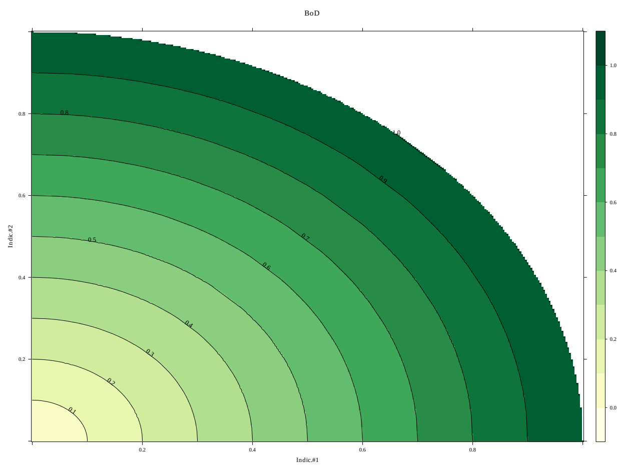

Figure 3: BoD

The main drawbacks are directly linked with the DEA problem solution: since the weights are unit specific, cross-unit comparisons are not possible and the values of the scoreboard depend on the benchmark performance.

There are also two other drawbacks: the multiplicity of equilibria (see subsection 3.1) and the robustness (see section 4).

3.1 Multiplicity of equilibria: Variance weighted BoD

Equation (8), in fact, hides a problem of the multiplicity of equilibria. Thus weights are not uniquely determined (even though the CI is unique). The weights values for the units are thus to be chosen among many (infinite) possibilities. It is also worth noting that multiple solutions are likely to depend upon the set of constraints imposed on the weights of the maximization problem: the wider the range of the variation of weights, the lower the possibility of obtaining a unique solution.

The optimization process could lead to many zero weights if no restrictions on the weights are imposed, so bounding restrictions on weights are necessary for this method to be of practical use.

Many additional weighting schema in recent years have been proposed; see for instance, Rogge Rogge (2012), ...

Following an original methodology, we have proposed (please see Mazziotta and Vidoli (2009)) to add a particular set of weight constraints[5] endogenously determined to account for the variability of each simple indicator, in terms of sample variance of each indicator.

[5] More precisely we added an "Assurance regions", type I (AR I), please see Thanassoulis et al. (2004).

Our basic thesis involves weighting simple indicators by their own sample variance; thus, indicators with a high variability will strongly affect the composite indicator. There are however consequences to this approach: our measurement has to be read as a âĂIJgap indicatorâĂİ among the unit characteristics. Our preliminary hypothesis is that every single indicator $I_q, q = 1...Q$ is a probabilistic variable, having a Normal Gaussian distribution[6]:

$$I_q \sim N(\mu_{I_q}, \sigma_{I_q}), \forall q = 1, ..., Q \tag{9}$$

In this way the vertical variance of each indicator can be computed in a standard probabilistic setting and the unbiased variance confidence interval is:

$$P(\frac{n-1}{\chi^2_{n-1,1-\alpha/2}}\overline{S}^2 < \sigma^2 < \frac{n-1}{\chi^2_{n-1,\alpha/2}}) = 1 - \alpha \tag{10}$$

$$P(low_{I_q} < \sigma^2 < high_{I_q}) = 1 - \alpha \tag{11}$$

Even when the underlying distribution is not Normal, the procedure is still used to obtain the approximate confidence bounds for the variance estimated. If the distribution is not too far from the Normal one, the procedure is sufficiently robust and usually works well. We can use low_{I_q} and $high_{I_q}$ for each indicator to reconstruct the marginal rates of substitution among indicators:

$$\frac{low_{I_i}}{high_{I_j}} \leq \frac{w_{I_i}}{w_{I_j}} \leq \frac{low_{I_j}}{high_{I_i}}, \forall i, j = 1, ..., Q \tag{12}$$

Contrasting the confidence interval inferior limit of the variance with the maximum of another one assumes a *"benefit of doubt"* attitude, by not imposing an exact relationship among weights, but thereby establishing a range in which every unit obtains the maximum relative weight.

3.2 Non-compensatory BoD: BoD-PCV

Munda et al.Munda and Nardo (2005) affirm the "*if one wants the weights to be interpreted as "importance coefficients" (or equivalently symmetrical importance of variables) non-compensatory aggregation procedures must be used*"[7].

To overcome the "non-compensatory" drawback, we can easily incorporate the DeMuro et al. (2010)'s idea in the basic BoD model assuming that each indicator may not be replaced by the others or is so only in part. According to this hypothesis, the method involves introducing a penalty for units that have not balanced a budget for all components, such as:

$$BoD\_PCV_i = BoD_i(1 - cv_i^2), \forall i = 1, ..., N \tag{13}$$

[6] To bypass this assumption, future developments of this methodology may involve the analysis of the Kernel Density Estimate of the simple indicators and their own sample variance.

[7] For a complete survey about compensatory and non-compensatory approaches, please see Vansnick (1990).

where cv_i^2 represents the coefficient of variation for the unit i between all indicators. This approach, therefore, allows for the penalization of the units that, while having an equal BoD score, have a greater imbalance among indicators.

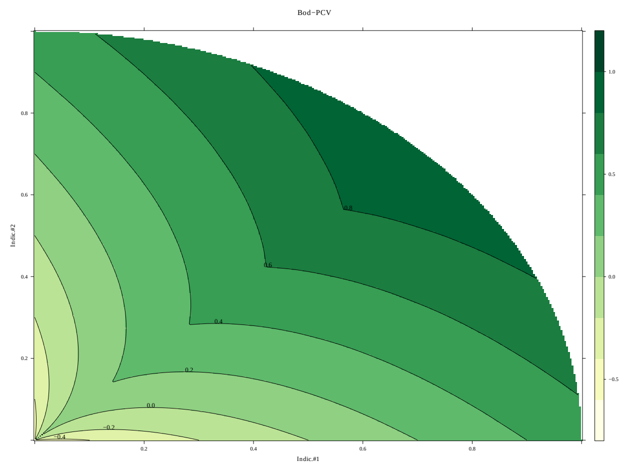

Figure 4: BoD-PCV

With respect to the individual BoD and MPCV approaches, BoD-PCV approach has two advantages: (i) it consistently takes into account the benchmark units on the frontier and, (ii) at the same time, it penalizes, in the case of non-compensatory issues, the presence of unbalanced data.

BoD-PCV model (see Figure 5 in which we added the presence of an outlier), is logically and strongly influenced by outliers in the simple indicators.

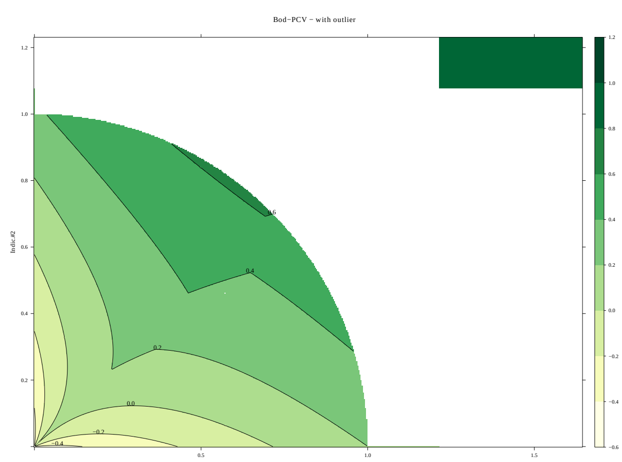

Figure 5: BoD-PCV in presence of outliers

4 Robust BoD (Order-m methods application to CI)

One of the main drawbacks of DEA/FDH nonparametric estimators is their sensitivity to extreme values and outliers. To introduce order-m we first expose the simplified idea and then we more precisely formalize the model.

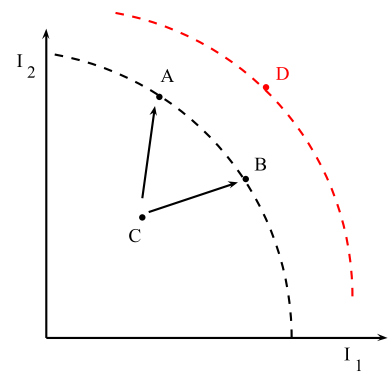

Figure 6: Presence of outliers in a frontier framework

In this context, Cazals et al. (2002) proposed a more robust nonparametric estimator of the frontier. It is based on the concept of the expected minimum input function of order-m.
Extending these ideas to the full multivariate case, Daraio and Simar (2005) defined the concept of the expected order-m input efficiency score. Daraio and Simar (2005) affirmed that: *"in place of looking for the lower boundary of the support of $F_X(x|y)$, as was typically the case for the full-frontier (DEA or FDH), the order-m efficiency score can be viewed as the expectation of the minimal input efficiency score of the unit (x, y), when compared to m units randomly drawn from the population of units producing more outputs than the level y.*

We extend Daraio and Simar (2005)'s idea into CI's framework by drawing repeatedly and with replacement m observations from the original sample of n observations and choosing only from those observations which are obtaining higher performance scores (I_1, I_2) - *red lines* - than the evaluated observation C.

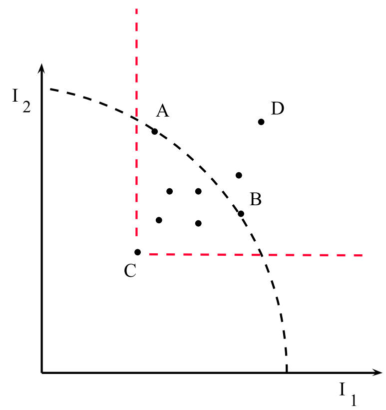

Figure 7: The support of unit C

In other words and practically speaking:

- We draw m observation only from those observations which are obtaining higher performance scores than the evaluated observation C;
- We label this set as SET_{bm};
- We estimate BoD scores relative to this subsample SET_{bm} for B times;
- Having obtained the B scores, we compute the arithmetic average.

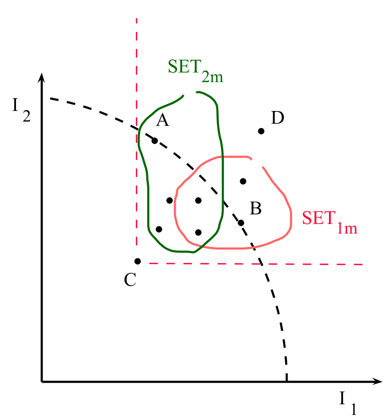

Figure 8: Subsamples

This is certainly a less extreme benchmark for the unit C than the *"absolute"* maximum achievable level of output.
C is compared to a set of m peers (potential competitors) producing more than

its level and we take as a benchmark, the expectation of the maximum achievable CI in place of the absolute maximum CI.

More accurately, Daraio and Simar (2005) propose (for a more complete theoretical summary see Fried and Lovell (2008) and Daraio and Simar (2007a)) a probabilistic formulation of efficiency concepts, supposing that the Data Generating Process (DGP) of (X, Y) is completely characterized by:

$$H_{XY}(x, y) = Prob(X \leq x, Y \geq y) \tag{14}$$

so $H_{XY}(x, y)$ can be interpreted as the probability for a unit operating at the level (x, y) to be dominated. Note that it is monotone nondecreasing with x and monotone non increasing with y. This joint probability can be decomposed as follows (yet in a input-oriented framework):

$$H_{XY}(x, y) = Prob(X \leq x|Y \geq y)Prob(Y \geq y) = S_{X|Y}(x|y)F_Y(y) \tag{15}$$

An input oriented efficiency score $\widehat{\theta}(x, y)$ for $(x, y) \in \Psi$ is defined for all y with $F_Y(y) > 0$ as

$$\widehat{\theta}(x, y) = \inf\{\theta|(\theta x_0, y_0) \in \Psi\} = \inf\{\theta|H(\theta x, y) > 0\} \tag{16}$$

Applying Daraio and Simar (2005)'s ideas to the particular case of composite indicators, equation (14) can be written:

$$H(x, y) = Prob(X \equiv 1, Y \geq y) \tag{17}$$

$$\text{where } \Psi \text{ is the support of } H(x, y) \tag{18}$$

So Farrel-Debreu (input) efficiency score, since $Prob(X \equiv 1|Y \geq y) = 1$ can be written as:

$$H(x, y) = Prob(X \equiv 1|Y \geq y)Prob(Y \geq y) = F_Y(y) \tag{19}$$

$$\theta(1, y_0) = \inf\{\theta|(\theta, y_0) \in \Psi\} = \inf\{\theta|H(\theta, y) > 0\} \tag{20}$$

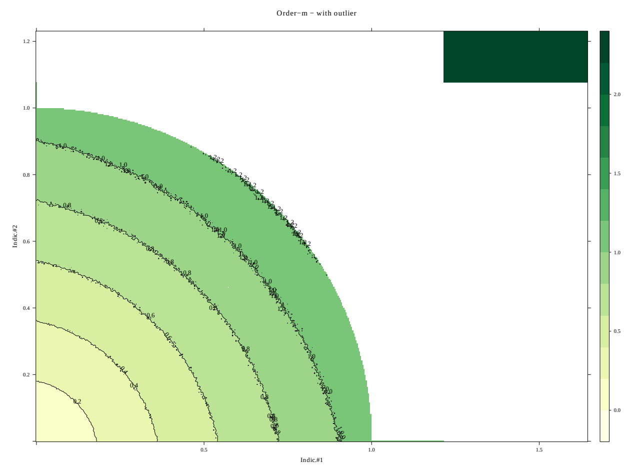

Figure 9: Order-m in presence of outliers

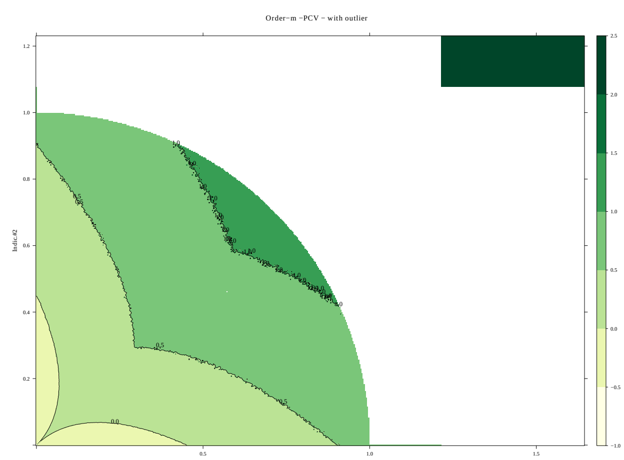

Figure 10: Order-m PCV in presence of outliers

5 Conditional robust BoD (Conditional order-m methods application to CI)

Using the probabilistic formulation, Cazals et al. (2002) also suggested a *conditional efficiency approach* which includes external environmental factors that might influence the production process but are neither inputs nor outputs under the control of the producer. Daraio and Simar (2005) extended these ideas to a more general multivariate setup and proposed a practical methodology to evaluate the effect of environmental variables in the production process.

Later, an extension to convex nonparametric models was proposed Daraio and Simar (2007b) and also a significant amount of works has been done to prove the consistency and the asymptotic properties of different conditional efficiency estimators (see Jeong et al. (2008)).

The conditional efficiency approach consists of conditioning (for simplicity, we report only univariate case) the production process to a given value of $Z = z$, where Z denote a variable characterizing the operational environment. The joint probability function given $Z = z$ can be defined as:

$$\begin{aligned} H_{XY|Z}(x,y) = &\quad Prob(X \leq x, Y \geq y|Z = z) \\ = &\quad Prob(X \leq x|Y \geq y, Z = z)Prob(Y \geq y|Z = z) \\ = &\quad S_{X|Y,Z}(x|y,z)F_Y(y|z) \end{aligned} \tag{21}$$

Daraio and Simar (2005), by analogy with the output Farrell efficiency score, define the conditional output efficiency measure:

$$\widehat{\theta}(x,y|z) = \inf\{\theta|(\theta x_0, y_0|z) \in \Psi\} = \inf\{\theta|H(\theta x, y|z) > 0\} \tag{22}$$

To reduce the deterministic nature, again instead of using the full support of $S(x|y,z)$ we can use the expected value of maximum output efficiency score of the unit (x, y), when compared to m units randomly drawn from the population of units for which $X \leq x$. Analogously to the unconditional order-m efficiencies, conditional efficiency measure $\lambda_m(x,y|z)$ can be expressed using the following integral:

$$\lambda_m(x,y|z) = \int_0^\infty [1 - (1 - S_x(ux|y,z))^m]du \tag{23}$$

Estimating $S_x(x|y,z)$ non parametrically is somewhat more difficult than for the unconditional case, as we need to use smoothing techniques in z (due to the equality constraint $Z = z$).

Witte and Kortelainen (2008) proposed to adapt the nonparametric conditional efficiency measures to include mixed (i.e. both continuous and discrete ordered and unordered) exogenous variables by specifying an appropriate kernel function which smooths the mixed variables.

The conditional output efficiency measure ($X \equiv 1$):

$$\widehat{\theta}(x,y|z) = \inf\{\theta|(\theta, y_0|z) \in \Psi\} = \inf\{\theta|H(\theta, y|z) > 0\} \tag{24}$$

6 Application

As already done in previous works (Mazziotta and Vidoli (2009); Mazziotta et al. (2010)), the proposed new approach concerning the construction of composite indicators has been applied to infrastructure endowment.

In this case we have limited the analysis to terrestrial transport infrastructure (roads and railways), for the following reasons: i) the relevance of this type of infrastructure in any regional development strategy; ii) the coherence with some previous applications (Mazziotta et al. (2010)); iii) the opportunity of following the same pattern shown in the methodological part of this paper, in particular in the figures 1 to 10, in which two dimensions (that is, two infrastructure categories) have been considered.

Because of the meaning of this paper is essentially methodological, the application and the relative results have a nature of empirical example, more than effective analysis of infrastructure endowment. In any case, we believe the obtained results are very interesting, in order to verify the robustness of the proposed methods concerning the construction of composite indicators. The following methods are considered: BoD, BoD-PCV, Order-m, Order-m PCV (for the meaning of these abbreviations, see previous paragraphs).

The application concerns two main categories of terrestrial transport: roads (separately, motorways and other roads) and railways (separately, electrified railways lines and railways lines double). Data set includes NUTS2 regions of the main European countries (in terms of population and GDP): France, Germany, Italy, Spain. In comparison with other works (Di Palma and Mazziotta (2003)) United Kingdom has been omitted, because of the lack of data concerning railways. Source of data base: Eurostat, Statistics by theme, 2012[8].

More specifically, we used the following basic indicators:

- Motorways - Kilometres per 1000 km^2;
- Other roads - Kilometres per 1000 km^2;
- Electrified railway lines Kilometres per 1000 km^2;
- Railway lines with double and more tracks - Kilometres per 1000 km^2.

and the simple indicators are been calculated as:

$$\begin{aligned} I_{Roads} =& \frac{2 \cdot \text{Motorways} + \text{Other roads}}{3} \\ I_{Trains} =& \frac{2 \cdot \text{Railway lines double} + \text{Electrified railway lines}}{3} \end{aligned} \tag{25}$$

[8]See http://epp.eurostat.ec.europa.eu/portal/page/portal/statistics/themes , Regional transport statistics (reg_tran)

The main results of the application (see Figures[9] 11, 12, 13 and 14 and Table 1) are the following ones:

- the four applied approaches produce four corresponding rankings of considered European regions. The Co-graduation Index (Spearman Index, see Table 1) shows the distance among such rankings. This distance is bigger in the case of comparison between the original BoD approach and the revised BoD approach by means of MPCV index. Evidently, the correction introduced in the construction of composite indicators by non-compensatory approach gives more changes in ranking than the correction caused by order-m approach;
- two regions always are on the top of the ranking: Berlin (the first region for railways) and Ile de France (the first region for roads). Their endowment level is always much more elevated in comparison of all the remaining areas: this situation produces a "crushing effect" that risks flattening the position of the other regions. The correction introduced by order-m approach mitigates this effect and allows to highlight the differences among the infrastructure levels of all the considered regions. From this point of view the order-m approach can be considered an improvement in regard to the other methods (BoD and MPCV);
- with reference to Italian regions, their ranking is not notably influenced by different approaches: Nord-Est is always in the first half position, Nord-Ovest and Centro are at the beginning of second half, and Sud and Isole are situated at the ending part of ranking.

Approaches	BoD	BoD-PCV	Order-m	Order-m PCV
BoD	1			
BoD-PCV	0.78	1		
Order-m	0.89	0.92	1	
Order-m PCV	0.70	0.93	0.93	1

Table 1: Spearman Index for different approaches of construction of composite indicators

[9] In order to highlight the correspondence of the obtained results with the methodological part of the paper, Figures 11, 12, 13 and 14 propose the same framework that we used in previous paragraphs. In fact, the results are represented by points corresponding to the infrastructure indicators in European regions, while the framework is only indicative of virtual pattern of each considered approach.

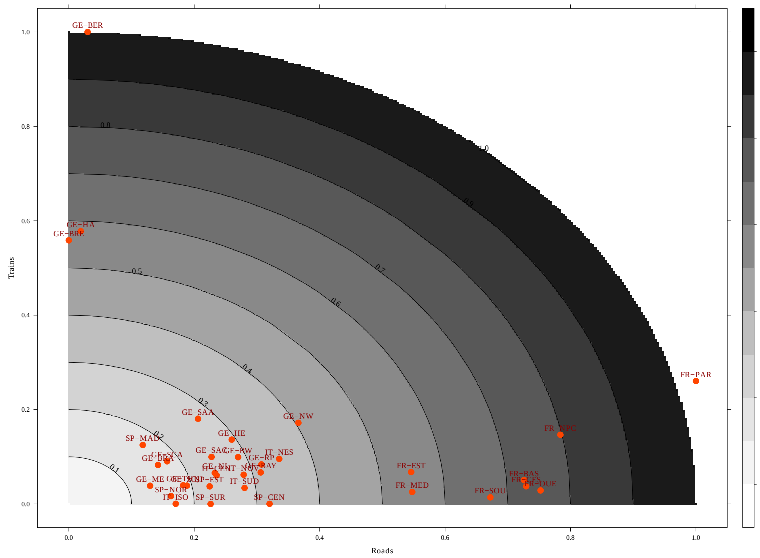

Figure 11: BoD Composite Indicator

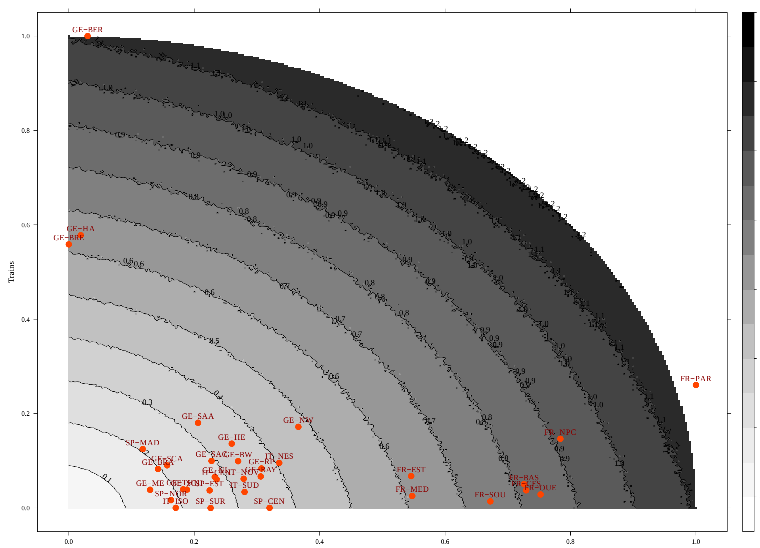

Figure 12: Order-m Composite Indicator

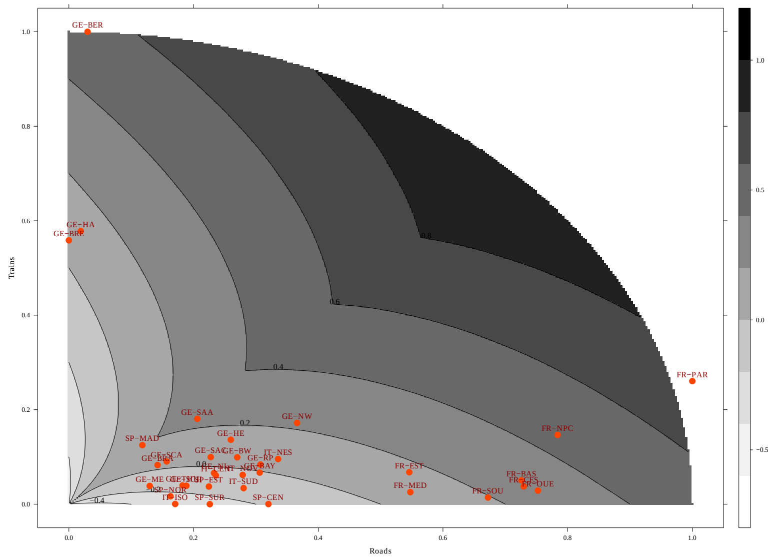

Figure 13: BoD-PCV Composite Indicator

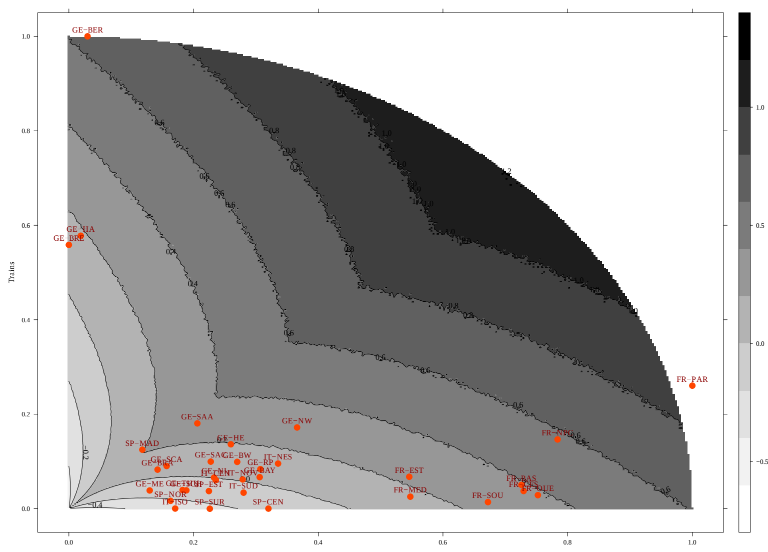

Figure 14: Order-m PCV Composite Indicator

Table 2 shows the complete results for NUTS2 regions.

	NUTS2	ABBR	Roads	Trains	BoD	Rank	Order-m	Rank	BoD PCV	Rank	Order-m PCV	Rank
1	Baden-Württemberg	GE-BW	0,269977	0,099161	0,3278	19	0,5922	16	0,0964	17	0,3608	14
2	Bayern	GE-BAY	0,306017	0,06684	0,3313	18	0,4933	20	0,0106	22	0,1726	23
3	Berlin	GE-BER	0,029957	1	1,0000	1	1,5877	1	0,5291	2	1,1168	2
4	Brandenburg	GE-BRA	0,142258	0,082735	0,2213	29	0,4529	24	0,0890	19	0,3206	17
5	Bremen	GE-BRE	0	0,558577	0,5656	9	0,9208	5	0,0656	20	0,4208	10
6	Hamburg	GE-HA	0,019104	0,577879	0,5970	8	1,1082	4	0,1290	13	0,6402	6
7	Hessen	GE-HE	0,259806	0,136359	0,3554	15	0,7533	12	0,1996	11	0,5975	8
8	Mecklenburg-Vorpommern	GE-ME	0,129653	0,03862	0,1710	34	0,2930	32	-0,0995	27	0,0226	27
9	Niedersachsen	GE-NI	0,232777	0,065871	0,2701	23	0,4589	23	-0,0093	23	0,1795	22
10	Nordrhein-Westfalen	GE-NW	0,366046	0,171969	0,4646	12	0,8723	8	0,2842	8	0,6920	5
11	Rheinland-Pfalz	GE-RP	0,307169	0,083764	0,3400	17	0,5324	19	0,0543	21	0,2466	20
12	Saarland	GE-SAA	0,206084	0,180603	0,3585	14	0,8786	6	0,3256	4	0,8456	4
13	Sachsen	GE-SAC	0,227573	0,099669	0,2980	22	0,5843	18	0,1026	16	0,3889	13
14	Sachsen-Anhalt	GE-SCA	0,156778	0,090421	0,2388	28	0,4818	21	0,1046	15	0,3475	16
15	Schleswig-Holstein	GE-SCH	0,188423	0,038911	0,2180	30	0,3190	30	-0,1108	30	-0,0099	29
16	Thüringen	GE-THU	0,182386	0,039589	0,2122	31	0,3340	29	-0,1094	29	0,0124	28

Table 2: European NUTS2 transport infrastructure by composite indicators (GE = Germany)

	NUTS2	ABBR	Roads	Trains	BoD	Rank	Order-m	Rank	BoD PCV	Rank	Order-m PCV	Rank
17	Noreste (ES)	SP-NOR	0,163031	0,016651	0,1935	33	0,2610	33	-0,2138	32	-0,1463	32
18	Comunidad de Madrid	SP-MAD	0,117844	0,124932	0,2434	27	0,6259	15	0,2288	9	0,6113	7
19	Centro (ES)	SP-CEN	0,320091	0,000266	0,3449	16	0,3952	27	-0,1543	31	-0,1039	31
20	Este (ES)	SP-EST	0,224599	0,037344	0,2529	26	0,3470	28	-0,1045	28	-0,0104	30
21	Sur (ES)	SP-SUR	0,226073	0	0,2543	25	0,3028	31	-0,2457	33	-0,1972	33
22	Île de France	FR-PAR	1	0,260499	1,0000	1	1,5488	2	0,7067	1	1,2555	1
23	Bassin Parisien	FR-BAS	0,725786	0,050461	0,7357	6	0,8766	7	0,3007	5	0,4416	9
24	Nord - Pas-de-Calais	FR-NPC	0,783837	0,146613	0,7917	3	1,2015	3	0,4493	3	0,8591	3
25	Est (FR)	FR-EST	0,545973	0,067611	0,5625	11	0,7430	13	0,1727	12	0,3532	15
26	Ouest (FR)	FR-OUE	0,752274	0,028697	0,7613	4	0,8703	10	0,2980	6	0,4071	12
27	Sud-Ouest (FR)	FR-SOU	0,672186	0,014044	0,6841	7	0,7886	11	0,2046	10	0,3091	19
28	Centre-Est (FR)	FR-CES	0,729351	0,037498	0,7392	5	0,8718	9	0,2881	7	0,4207	11
29	Méditerranée	FR-MED	0,547672	0,025485	0,5641	10	0,6655	14	0,1086	14	0,2099	21
30	Nord-Ovest	IT-NOV	0,278808	0,061891	0,3051	21	0,4675	22	-0,0132	24	0,1491	24
31	Nord-Est	IT-NES	0,33554	0,095591	0,3713	13	0,5876	17	0,0930	18	0,3093	18
32	Centro (IT)	IT-CEN	0,235669	0,060786	0,2674	24	0,4107	26	-0,0276	25	0,1157	25
33	Sud	IT-SUD	0,280233	0,033962	0,3065	20	0,4308	25	-0,0854	26	0,0389	26
34	Isole	IT-ISO	0,170493	0,000378	0,2007	32	0,2432	34	-0,2971	34	-0,2546	34

Table 3: European NUTS2 transport infrastructure by composite indicators (SP = Spain, FR= France, IT= Italy)

7 Final remarks

In this paper we have presented some new approaches in order to construct composite indicators. In particular, two aims have been pursued: i) correction of BoD Index by means a non-compensatory approach; ii) introduction of order-m approach as a more robust estimator in the field of nonparametric frontier techniques.

In order to the first aim, we have attempted to integrate BoD Index by a non-compensatory approach, introducing in the construction of composite indicators a penalty for unbalanced simple indicators. The resulting approach (BoD-PCV) presents two advantages: it takes in account the benchmark units on the frontier (peculiarity of BoD), and, at the same time, penalizes the presence of the unbalanced simple indicators (peculiarity of MPCV).

In order to the second aim, we have introduced in the construction of composite indicators the concept of the expected minimum input function of order-m, relevant overall in case of presence of outliers in a frontier framework. This approach has been applied with reference to both synthesis methods above indicated (BoD and Bod-PCV), obtaining a more robust estimation of composite indicators.

Finally, these approaches have been tested with reference to infrastructure endowment in European regions (terrestrial transport). The obtained results confirm the improvement in the robustness of composite indicators by the introduction of order-m technique.

References

Cazals, C., J. Florens, and L. Simar (2002). Nonparametric frontier estimation: A robust approach. *Journal of Econometrics 106 (1)*, 1–25. [Cited at page 9, 12]

Charnes, A., W. Cooper, and W. Rhodes (1978). Measuring the efficiency of decision making units. *European journal of operational research 2(4)*, 429 – 444. [Cited at page 5]

Cherchye, L. and T. Kuosmanen (2002). Benchmarking sustainable development: A synthetic metaindex approach. Technical report, EconWPA, Others 0210001. [Cited at page 5]

Cherchye, L., K. Lovell, W. Moesen, and T. V. Puyenbroeck (2005). One market, one number? a composite indicator assessment of eu internal market dynamics. Technical report, Working Paper Series ces0513, Katholieke Universiteit Leuven, Centrum voor Economische Studien. [Cited at page 2]

Cherchye, L., W. Moesen, and T. Puyenbroeck (2004). Legitimately diverse, yet comparable: On synthesizing social inclusion performance in the eu. *Journal of Common Market Studies 42(5)*, 919 – 955. [Cited at page 2]

Cherchye, L., E. Ooghe, and T. Van Puyenbroeck (2008). Robust human development rankings. *Journal of Economic Inequality 6*, 287–321. 10.1007/s10888-007-9058-8. [Cited at page 2]

Daraio, C. and L. Simar (2005). Introducing environmental variables in nonparametric frontier models: a probabilistic approach. *Journal of Productivity Analysis 24(1)*, 93–121. [Cited at page 9, 11, 12, 13]

Daraio, C. and L. Simar (2007a). *Advanced Robust and Nonparametric Methods in Efficiency Analysis*. Springer. [Cited at page 11]

Daraio, C. and L. Simar (2007b). Conditional nonparametric frontier models for convex and nonconvex technologies: A unifying approach. *Journal of Productivity Analysis 28*, 13–32. [Cited at page 13]

DeMuro, P., M. Mazziotta, and A. Pareto (2010). Composite indices of development and poverty: An application to mdgs. *Social Indicators Research 104(1)*, 1–18. [Cited at page 2, 7]

Deprins, D., L. L. Simar, and H. Tulkens (1984). *The Performance of Public Enterprizes: Concepts and Measurements*, Chapter Measuring Labor Inefficiency in Post Offices, pp. 243 – 267. North-Holland. [Cited at page 5]

Despotis, D. (2005a). Measuring human development via data envelopment analysis: the case of asia and the pacific. *Omega 33*(5), 385 – 390. OR and its Applications. [Cited at page 2]

Despotis, D. K. (2005b). A reassessment of the human development index via data envelopment analysis. *Journal of the Operational Research Society 56*, 969 – 980. [Cited at page 2]

Di Palma, M. and C. Mazziotta (2003). Infrastrutture, competitivita' e sviluppo: il caso italiano. *Economia italiana 1.* [Cited at page 14]

Freudenberg, M. (2003). Composite indicators of country performance: A critical assessment. Technical report, OECD Science, Technology and Industry Working Papers 2003/16, OECD, Directorate for Science, Technology and Industry. [Cited at page 2]

Fried, H. O. and C. A. K. Lovell (2008). *The Measurement of Productive Efficiency and Productivity Growth.* Oxford University Press, USA. [Cited at page 3, 11]

Jeong, S., B. Park, and L. Simar (2008). Nonparametric conditional efficiency measures: Asymptotic properties. *Annals of Operations Research Forthcoming.* [Cited at page 13]

Mahlberg, B. and M. Obersteiner (2001). Remeasuring the hdi by data envelopment analysis. Technical report, IIASA, Interim Report IR-01-069, Laxemburg, Austria. [Cited at page 2]

Mazziotta, C., M. Mazziotta, A. Pareto, and F. Vidoli (2010). La sintesi di indicatori territoriali di dotazione infrastrutturale: metodi di costruzione e procedure di ponderazione a confronto. *Rivista di Economia e Statistica del Territorio 1.* [Cited at page 14]

Mazziotta, C. and F. Vidoli (2009). La costruzione di un indicatore sintetico ponderato. un'applicazione della procedura benefit of the doubt al caso della dotazione infrastrutturale in italia. *Scienze Regionali 8*(1), 35–69. [Cited at page 6, 14]

Munda, G. and M. Nardo (2005). Constructing consistent composite indicators: the issue of weights. Technical report, EUR 21834 EN, European Commission. [Cited at page 7]

Nardo, M., M. Saisana, A. Saltelli, S. Tarantola, A. Hoffman, and E. Giovannini (2005). Handbook on constructing composite indicators: Methodology and user guide. *OECD Statistics Working Papers 2005/3, OECD, Statistics Directorate*. [Cited at page 2, 3]

Rogge, N. (2012). Undesirable specialization in the construction of composite policy indicators: The environmental performance index. *Ecological Indicators 23*, 143 – 154. [Cited at page 6]

Storrie, D. and H. Bjurek (2000). Benchmarking european labour market performance with efficiency frontier techniques. Technical report, CELMS Discussion papers, Goteborg University. [Cited at page 2]

Thanassoulis, E., M. C. Portela, and R. Allen (2004). Incorporating value judgments in dea. In W. W. Cooper, L. M. Seiford, and J. Zhu (Eds.), *Handbook on Data Envelopment Analysis*, Volume 71 of *International Series in Operations Research & Management Science*, pp. 99–138. Springer US. [Cited at page 6]

Vansnick, J. (1990). *Measurement theory and decision aid*, Chapter Readings in multiple criteria decision aid, pp. 81–100. Springer-Verlag, Berlin. [Cited at page 7]

Witte, K. D. and M. Kortelainen (2008). Blaming the exogenous environment? conditional efficiency estimation with continuous and discrete environmental variables. Technical report, Working Paper Series ces0833, Katholieke Universiteit Leuven, Centrum voor Economische Studien. [Cited at page 13]

Witte, K. D. and N. Rogge (2009). Accounting for exogenous influences in a benevolent performance evaluation of teachers. Technical report, Working Paper Series ces0913, Katholieke Universiteit Leuven, Centrum voor Economische Studien. [Cited at page 1, 5]